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DEFORMABILITY MODULUS OF JOINTED ROCKS, LIMITATION OF EMPIRICAL METHODS AND INTRODUCING A NEW ANALYTICAL APPROACH

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ABSTRACT: Deformability modulus of jointed rocks is a key parameter for stability analysis of underground structures by numerical modelling techniques. Intact rock strength, rock mass blockiness (shape and size of rock blocks), surface condition of discontinuities (shear strength of discontinuities) and confining stress level are the key parameters controlling deformability of jointed rocks. Considering cost and limitation of field measurements to determine deformability modulus, empirical equations which were mostly developed based on rock mass classifications are too common in practice. All well-known empirical formulations dismissed the impact of stress on deformability modulus. Therefore, these equations result in the same value for a rock at different stress fields. This paper discusses this issue in more detail and highlights shortcomings of existing formulations. Finally it presents an extension to analytical techniques to determine the deformability modulus of jointed rocks by a combination of the geometrical properties of discontinuities and elastic modulus of intact rock. In this extension, the effect of confining stress was incorporated in the formulation to improve its reliability.

INTRODUCTION

The deformability modulus of jointed rock mass is key parameters which is required for numerical and analytical analysis of structures in or on the rock mass. It is defined as the ratio of stress to corresponding strain (Figure 1) during loading of a rock mass, including elastic and inelastic behaviour (Ulusay and Hudson 2006).

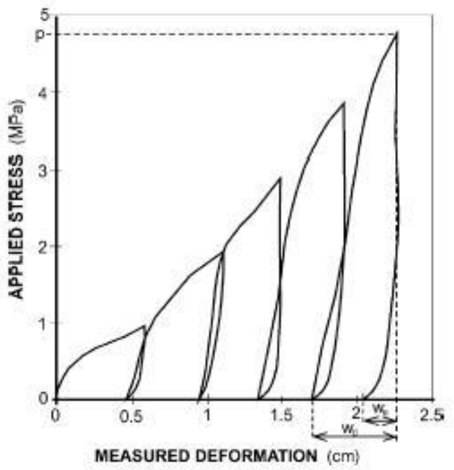

Figure 1: Typical loading response of jointed rock

Estimation of deformability modulus is a significant challenge for rock engineers. Although field tests and measurement are better methods to determine this parameter, they are costly and imply notable operational difficulties. In the most practical applications, empirical equations which developed on the basis of case studies and rock mass classification systems are common tools to estimate deformability modulus. Over the years, many empirical equations were introduced by researchers and engineers by

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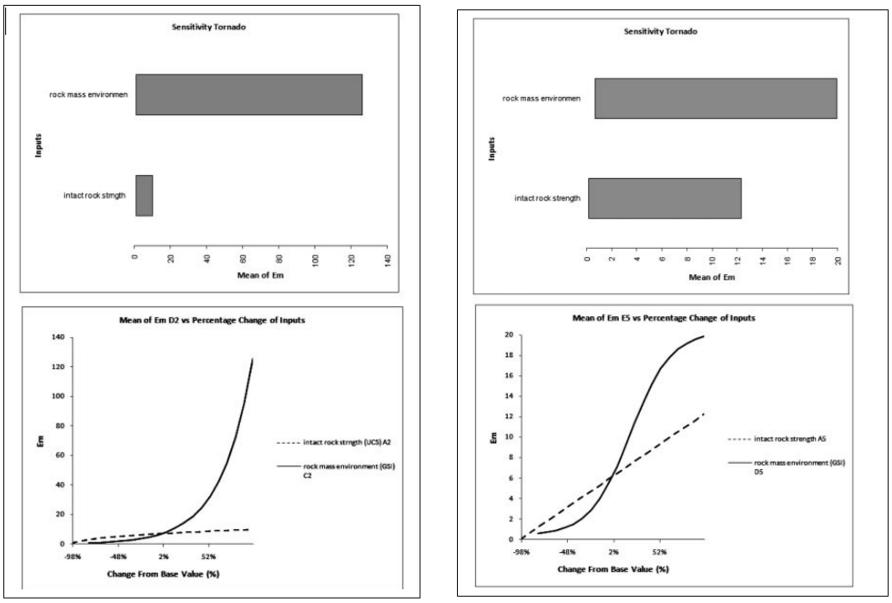
correlating field measurements with well-known classification systems such as RMR, Q, GSI, RMi, and so on. In Table 1, a list of these equations was presented.

Table 1: Empirical equations using RMR and GSI (Shen *et al.,* 2012)

Input parameters	Empirical equations	
Group 1 RMR	Bieniawski (1978)	$E_m = 2\text{RMR} - 100, \text{RMR} > 50$
	Serafim and Pereira (1983)	$E_m = 10^{(\text{RMR-10})/40}$
	Mehrotra (1992)	$E_m = 10^{(\text{RMR-20})/38}$
	Read et al. (1999)	$E_m = 0.1(\text{RMR}/10)^3$
Group 2 RMR and E_i	Nicholson and Bieniawski (1990)	$E_m = 0.01E_i(0.0028RMR^2 + 0.9e^{\frac{RMR}{22.83}})$
	Mitri et al. (1994)	$E_m = E_i[0.5(1 - (\cos(\pi\text{RMR}/100)))]$
	Sonmez et al. (2006)	$E_m = E_i 10^{((\text{RMR-100})(100\text{-RMR}))/(4000\exp(-\text{RMR}/100))}$
Group 3 GSI and D	Hoek et al. (2002)	$E_m = (1 - 0.5D)10^{\left(\frac{\text{GSI}-10}{40}\right)}, \sigma_{\text{ci}} > 100\text{ MPa}$
	Hoek and Diederichs (2006)	$E_m(\text{MPa}) = 10^5\left(\frac{1-0.5D}{1+e^{((75+25D-\text{GSI})/11)}}\right)$
Group 4 GSI, D and E_i	Carvalho (2004)	$E_m = E_i(s)^{0.25}, \quad s = \exp\left(\frac{\text{GSI}-100}{9-3D}\right)$
	Sonmez et al. (2004)	$E_m = E_i(s^a)^{0.4}, \quad s = \exp\left(\frac{\text{GSI}-100}{9-3D}\right) \quad a = 0.5 + \frac{1}{6}(e^{-\text{GSI}/15} - e^{-20/3})$
	Hoek and Diederichs (2006)	$E_m = E_i\left(0.02 + \frac{1-0.5D}{1+e^{((60+15D-\text{GSI})/11)}}\right)$
Group 5 GSI, D and σ_{ci}	Hoek and Brown (1997)	$E_m = \sqrt{\frac{\sigma_{\text{ci}}}{100}}10^{\left(\frac{\text{GSI}-10}{40}\right)}$
	Hoek et al. (2002)	$E_m = (1 - 0.5D)\sqrt{\frac{\sigma_{\text{ci}}}{100}}10^{\left(\frac{\text{GSI}-10}{40}\right)}, \sigma_{\text{ci}} \leqslant 100\text{ MPa}$
	Beiki et al. (2010)	$E_m = \tan\left(\sqrt{1.56 + (\ln(\text{GSI}))^2}\right)\sqrt[3]{\sigma_{ci}}$

Deformability of jointed rocks is controlled by deformation of its intact rock blocks and discontinuities (Hoek and Brown 1997). Zoorabadi (2010) performed a parameter study on some of the existing empirical equations to explore the contribution of intact rock and rock mass condition to the deformability modulus estimated from those equations. It was found that in Hoek and Brown (1997) equation, intact rock properties (UCS) has a small contribution to the rock mass modulus. This condition was modified in Hoek and Diederichs (2006) equation (the most common equation in practice) which gives more contribution for intact rock property (Figure 2a, b).



a b

Figure 2: Parameter study on Hoek and Brown (1997) and Hoek and Diederichs (2006) equations (Zoorabadi 2010)

Stress dependency of deformability modulus which was not considered in empirical equation is the main shortcoming of all these equation and is the main objective of this paper. Deformability of rock discontinuities and rotation of rock block have a significant influence on deformability of jointed rocks located at ground surface where stress level in negligible. An applied normal stress on a rock fracture causes the fracture to close and decreases the aperture. The deformability of rock fractures due to normal stress has been studied intensively by several researchers. Goodman (1976) performed laboratory tests and found a significant nonlinear relationship between applied stress and fracture closure. He also found that the nonlinear trend approaches an asymptote at high stress values (Figure 3).

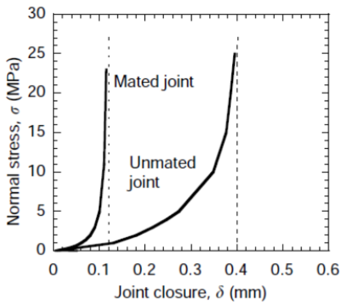


Figure 3: Fracture closure due to applied normal stress (Goodman 1976)

Therefore, deformability of rock mass containing discontinuities would have different values at different depth or stress fields. In this paper an analytical method was applied to assess the variation of deformability modulus with depth and to investigate the contribution intact rock properties have on deformation modulus of rock masses.

ANALYTICAL FORMULATION TO ESTIMATE DEFORMABILITY MODULUS OF ROCK MASS

Analytical formulations use the elastic behaviour concept and combined mechanical properties and geometrical characteristics of rock discontinuities to determine the deformability modulus. Li (2001) used superposition principle and introduced an analytical approach to determine deformability modulus of a block of rock mass containing single or multiple joint sets. For multiple joint sets, his formulation for deformability modulus in loading direction has the following form:

$$\frac{1}{E'} = \frac{1}{E} + \sum_{i=1}^{N} \frac{\cos^2\theta_i}{S_i}\left(\frac{1}{k_{ni}}\cos^2\theta_i + \frac{1}{k_{si}}\sin^2\theta_i\right) \quad (1)$$

where, E' is deformability modulus in loading direction, E is elastic modulus of intact rock, k_{ni} presents the normal stiffness of ith joint set, k_{si} is shear stiffness of ith joint set, S_i is spacing of ith joint set, N present the number of joint sets, and θ_i is the angle between loading direction and normal vector of ith joint set (Figure 4).

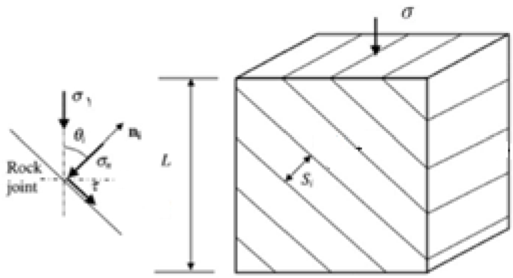


Figure 4: A block of rock containing a single joint set loaded by uniaxial stress condition (modified from Ebadi et al., 2011)

As it was shown in Figure 3, normal stiffness represents the rate of change in normal stress with respect to discontinuity closure. Bandis *et al.,* (1983) proposed the following empirical equation to estimate the normal stiffness of discontinuity under normal stress of σ_n as follows:

$$k_n = k_{ni}\left(1 - \frac{\sigma_n}{V_m k_{ni} + \sigma_n}\right)^{-2} \quad (2)$$

where, k_{ni} is initial normal stiffness, $\sigma_n$is current level of applied normal stress, and V_m is maximum closure of fracture. The ollowing equation was introduced to calculate initial normal stiffness on the basis of Joint Roughness Coefficient (JRC), Joint Compression Strength (JCS) and initial aperture (a_j) of fracture as follows:

$$k_{ni} = -7.15 + 1.75JRC + 0.02(\frac{JCS}{a_j}) \quad (3)$$

The empirical equation to calculate the initial aperture of fracture has the following form:

$$a_j = \frac{JRC}{5}(0.2\frac{\sigma_c}{JCS} - 0.1) \quad (4)$$

where, σ_c represents the uniaxial compression strength of rock. Maximum closure of fracture is function of loading cycle and can be estimated from following empirical equation:

$$V_m = A + B(JRC) + C(\frac{JCS}{a_j})^D \quad (5)$$

Constant values of A, B, C and D for each cycle of loading are listed in Table (2).

Table 2: Constant values to estimate the maximum closure of rock fracture (Bandis *et al.,* 1983)

Constant	1st Cycle	2nd Cycle	3rd Cycle
A	-0.2960 ± 0.1258	-0.1005 ± 0.0530	-0.1032 ± 0.0680
B	-0.0056 ± 0.0022	-0.0073 ± 0.0031	-0.0074 ± 0.0039
C	2.2410 ± 0.3504	1.0082 ± 0.2351	1.1350 ± 0.3261
D	-0.2450 ± 0.1086	-0.2301 ± 0.1171	-0.2510 ± 0.1029

Now, combination of Equations of 1 to 5 provide the capability to calculate the deformability modulus of jointed rocks when geometrical properties of discontinuities and field stress information are known. A real case located at Eastern of Australia (NSW) was used to investigate the impact of depth on the deformability modulus of jointed rock mass. In this case, the photogrammetry technique was implemented to determine the geometrical properties of discontinuities. In Figure 5 a photogrammetry results was presented.

The orientation and spacing of rock discontinuities for this case are listed in Table 3. To do a parameter study for this case, JRC and JCS of discontinuities were assumed as shown in Table 3. The elastic modulus of 16 GPa and UCS of 40 MPa were supposed for intact rock.

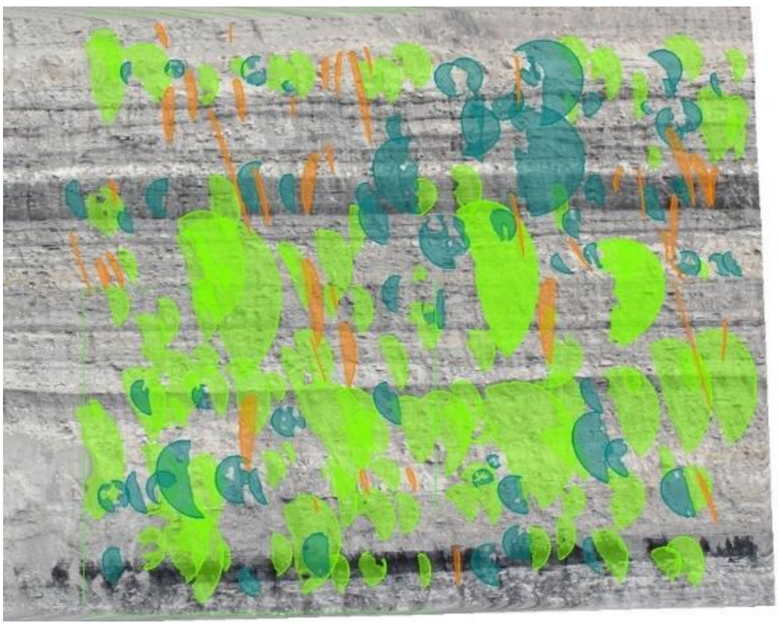


Figure 5: Photogrammetry image with detected defects

Table 3: Geometrical properties of rock discontinuities

Joint set	Dip	Dip/Dir	Spacing [m]	JRC	JCS
A	85	113	2.03	13	30
B	64	41	1.77	13	30
C	80	331	3.83	13	30
Bedding plane	24	156	4	10	30

Overcoring measurements and analysis of acoustic scanner data indicate a strong NW orientation for maximum horizontal stress. Furthermore, these tests show that the average ratio between maximum horizontal stress and minimum horizontal stress is $\sigma_H/\sigma_h = 1.5$. Since these tests had been performed at a limited number of depths, it was not possible to use them to estimate the magnitude of field stress variation with depth. Nemcik *et al.,* (2005) presented statistical analysis of measured stress in Australia coal mines as Figure (6). The most probable trend in this dataset was used to determine the ratio between maximum horizontal stress and vertical stress at different depth. Magnitude and orientation of the maximum horizontal stress was considered as stress level that controls the normal and shear stiffness of discontinuities for this case.

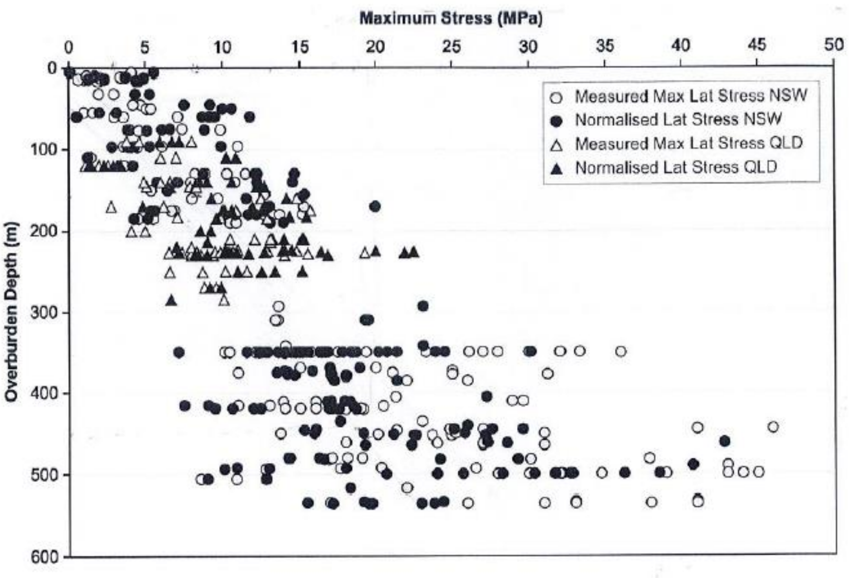


Figure 6: Maximum horizontal stress versus depth (Nemcik *et al.,* 2005).

The magnitude of the maximum horizontal stress was determined from Figure 6 and the angle between this stress component and discontinuities was used for calculations in Equation 1. In Figure 7, variation of calculated deformability modulus with depth is presented.

The deformability modulus of a block containing detected discontinuities at the ground surface (zero acting normal stress was assumed) was calculated to be 7.2 GPa. This value is around 0.45% of elastic modulus of intact rock and demonstrates the controlling impact of discontinuity deformation on deformability of a jointed rock. Deformability modulus of a this case increases significantly with depth increase. As it can be seen, just at 50 m depth, it would have a magnitude of 12.5 GPa which is 0.78% of the elastic modulus of intact rock. For depths deeper that 200 m, deformability modulus of a this rock mass would be more that 90% of the elastic modulus of intact rock. These results highlight a decreasing trend for discontinuity influence on the deformability of jointed rocks when depth increases.

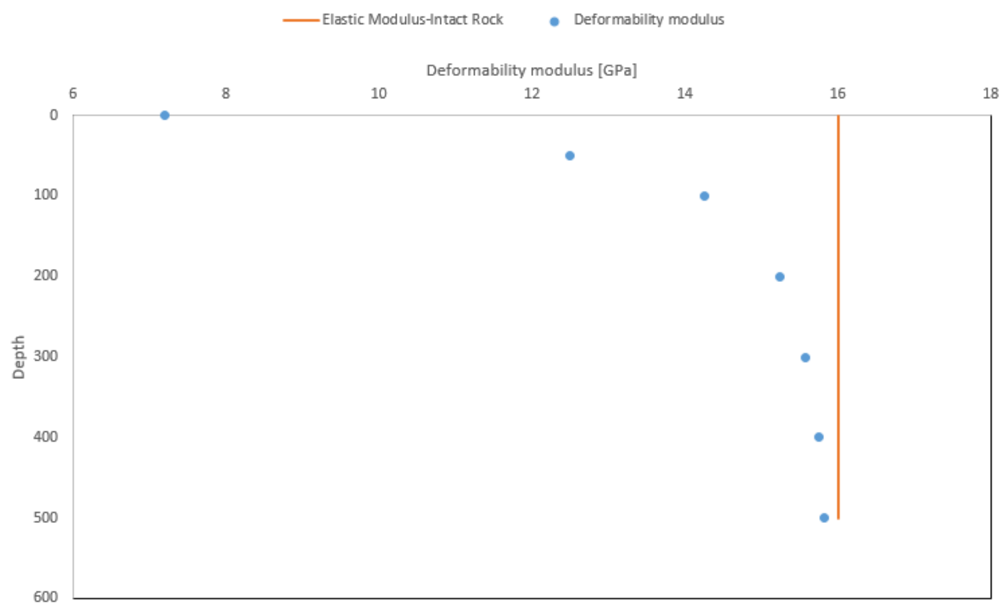


Figure 7: Variation of deformability modulus with depth

With available data for spacing of discontinuities (S), the method introduced by Snomez and Ulusay (1999) can be used to determine the GSI of jointed rock for this case (Figure 8).

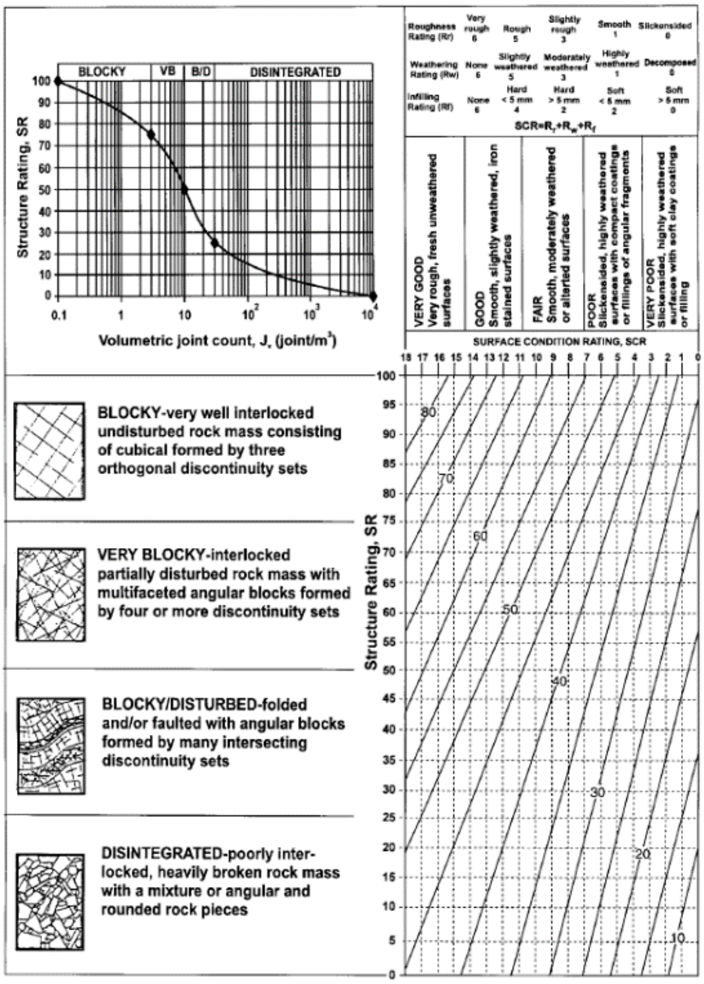


Figure 8: GSI table (Snomez and Ulusay 1999).